

# Time Reversal Invariance Violation in Neutron Deuteron Scattering

Young-Ho Song,<sup>1,\*</sup> Rimantas Lazauskas,<sup>2,†</sup> and Vladimir Gudkov<sup>1,‡</sup>

<sup>1</sup>*Department of Physics and Astronomy,*

*University of South Carolina, Columbia, SC, 29208*

<sup>2</sup>*IPHC, IN2P3-CNRS/Université Louis Pasteur BP 28,*

*F-67037 Strasbourg Cedex 2, France*

(Dated: January 19, 2013)

## Abstract

Time reversal invariance violating (TRIV) effects for low energy elastic neutron deuteron scattering are calculated for meson exchange and EFT-type of TRIV potentials in a Distorted Wave Born Approximation, using realistic hadronic strong interaction wave functions, obtained by solving three-body Faddeev equations in configuration space. The relation between TRIV and parity violating observables are discussed.

PACS numbers: 24.80.+y, 25.10.+s, 11.30.Er, 13.75.Cs

---

\* song25@mailbox.sc.edu

† rimantas.lazauskas@ires.in2p3.fr

‡ gudkov@sc.edu

## I. INTRODUCTION

A search for Time Reversal Invariance Violation (TRIV) in nuclear physics has been a subject of experimental and theoretical investigation for several decades. It has covered a large variety of nuclear reactions and nuclear decays with T-violating parameters which are sensitive to either CP-odd and P-odd (or T- and P-violating) interactions or T-violating P-conserving (C-odd and P-even) interactions. There are a number of advantages of the search for TRIV in nuclear processes. The main advantage is the possibility of enhancement of T-violating observables by many orders of a magnitude due to complex nuclear structure (see, i.e. paper [1] and references therein). Another advantage to be mentioned is the availability of many systems with T-violating parameters which provides assurance to have enough observations against possible “accidental” cancellation of T-violating effects due to unknown structural factors related to strong interactions. Taking into account that different models of CP-violation may contribute differently to a particular T/CP-observable<sup>1</sup>, which may have unknown theoretical uncertainties, TRIV nuclear processes shall provide complementary information to electric dipole moments (EDM) measurements.

One promising approach for a search for TRIV in nuclear reactions is a measurement of TRIV effects in transmission of polarized neutron through polarized target. These effects could be measured at new spallation neutron facilities, such as the SNS at the Oak Ridge National Laboratory or the J-SNS at J-PARC, Japan. It was shown that these TRIV effects can be enhanced [2] by a factor of  $10^6$ . Similar enhancement factors have been observed for parity violating effects in neutron scattering. In contrast to the parity violating (PV) case, the enhancement of TRIV effects lead not only to the opportunity to observe T violation, but also to select models of CP-violation based on the values of observed parameters. However, existing estimates of CP-violating effects in nuclear reactions have at least one order of magnitude of accuracy, or even worse. In this relation, it is interesting to compare the calculation of TRIV effects in complex nuclei with the calculations of these effects in simplest few body systems, which could be useful for clarification of influence of nuclear structure on values of TRIV effects. Therefore, as a first step to many body nuclear effects, we study TRIV and parity violating effects in one of the simplest available nuclear process, namely

---

<sup>1</sup> For example, QCD  $\theta$ -term can contribute to neutron EDM, but cannot be observed in  $K^0$ -meson decays.

On the other hand, the CP-odd phase of Cabibbo-Kobayashi-Maskawa matrix was measured in  $K^0$ -meson decays, but its contribution to neutron EDM is extremely small and beyond the reach with the current experimental accuracy.

elastic neutron-deuteron scattering.

We treat TRIV nucleon-nucleon interactions as a perturbation, while non-perturbed three-body wave functions are obtained by solving Faddeev equations for realistic strong interaction Hamiltonian, based on AV18+UIX interaction model. For description of TRIV potentials, we use both meson exchange model and effective field theory (EFT) approach.

## II. OBSERVABLES

We consider TRIV and PV effects related to  $\boldsymbol{\sigma}_n \cdot (\mathbf{p} \times \mathbf{I})$  correlation, where  $\boldsymbol{\sigma}_n$  is the neutron spin,  $\mathbf{I}$  is the target spin, and  $\mathbf{p}$  is the neutron momentum, which can be observed in the transmission of polarized neutrons through a target with polarized nuclei. This correlation leads to the difference [3] between the total neutron cross sections for  $\boldsymbol{\sigma}_n$  parallel and anti-parallel to  $\mathbf{p} \times \mathbf{I}$ , which is

$$\Delta\sigma_{T\mathbf{p}} = \frac{4\pi}{p} \text{Im}(f_+ - f_-), \quad (1)$$

and neutron spin rotation angle [4]  $\phi$  around the axis  $\mathbf{p} \times \mathbf{I}$

$$\frac{d\phi_{T\mathbf{p}}}{dz} = -\frac{2\pi N}{p} \text{Re}(f_+ - f_-). \quad (2)$$

Here,  $f_{+,-}$  are the zero angle scattering amplitudes for neutrons polarized parallel and anti-parallel to the  $\mathbf{p} \times \mathbf{I}$  axis, respectively,  $z$  is the target length, and  $N$  is a number of target nuclei per unit volume. It should be noted that these two parameters cannot be simulated by final state interactions (see, for example [1] and references therein), therefore, their measurements are an unambiguous test of violation of time reversal invariance similar to the case of neutron electric dipole moment.

The scattering amplitudes can be represented in terms of matrix  $\hat{R}$  which is related to scattering matrix  $\hat{S}$  as  $\hat{R} = \hat{1} - \hat{S}$ . We define matrix element  $R_{l'\mathcal{S}',l\mathcal{S}}^J = \langle l'\mathcal{S}' | R^J | l\mathcal{S} \rangle$ , where unprimed and primed parameters correspond to initial and final states,  $l$  is an orbital angular momentum between neutron and deuteron,  $\mathcal{S}$  is a sum of neutron spin and deuteron total angular momentum, and  $J$  is the total angular momentum of the neutron-deuteron system. For low energy neutron scattering, one can consider only  $s$ - and  $p$ -wave contributions, which leads to the following expressions for the TRIV parameters

$$\frac{1}{N} \frac{d\phi_{T\mathbf{p}}}{dz} = -\frac{\pi}{2p^2} \text{Re} \left[ \sqrt{2} R_{0\frac{1}{2},1\frac{3}{2}}^{\frac{1}{2}} - \sqrt{2} R_{1\frac{3}{2},0\frac{1}{2}}^{\frac{1}{2}} + 2R_{0\frac{3}{2},1\frac{1}{2}}^{\frac{3}{2}} - 2R_{1\frac{1}{2},0\frac{3}{2}}^{\frac{3}{2}} \right], \quad (3)$$

$$\Delta\sigma_{T\vec{p}} = \frac{\pi}{p^2} \text{Im} \left[ \sqrt{2} R_{0\frac{1}{2},1\frac{3}{2}}^{\frac{1}{2}} - \sqrt{2} R_{1\frac{3}{2},0\frac{1}{2}}^{\frac{1}{2}} + 2R_{0\frac{3}{2},1\frac{1}{2}}^{\frac{3}{2}} - 2R_{1\frac{1}{2},0\frac{3}{2}}^{\frac{3}{2}} \right]. \quad (4)$$

The symmetry violating  $\hat{R}$ -matrix elements can be calculated with a high level of accuracy in Distorted Wave Born Approximation (DWBA) as

$$R_{l'\mathcal{S}',l\mathcal{S}}^J \simeq 4i^{-l'+l+1} \mu p^{(-)} \langle \Psi, (l'\mathcal{S}')JJ^z | V_{T\vec{p}} | \Psi, (l\mathcal{S})JJ^z \rangle^{(+)}, \quad (5)$$

where  $\mu$  is a neutron-deuteron reduced mass,  $V_{T\vec{p}}$  is TRIV nucleon-nucleon potential, and  $|\Psi, (l'\mathcal{S}')JJ^z\rangle^{(\pm)}$  are solutions of 3-body Faddeev equations in configuration space for strong interaction Hamiltonian satisfying outgoing (incoming) boundary condition. The factor  $i^{-l'+l}$  in this expression is introduced to match the  $R$ -matrix definition in the modified spherical harmonics convention [5] with the wave functions in spherical harmonics convention used for wave-functions calculations. The matrix elements of TRIV potential in spherical harmonics convention are symmetric and  $R$ -matrix in modified spherical harmonics convention is antisymmetric under the exchange between initial and final states.

For calculations of wave-functions, we used jj-coupling scheme instead of  $l\mathcal{S}$  coupling scheme. We can relate  $R$ -matrix elements in  $l\mathcal{S}$  coupling scheme to jj-coupling scheme using unitary transformation (see, for example [6])

$$\begin{aligned} |[l_y \otimes (s_k \otimes j_x)_{\mathcal{S}}]_{JJ_z}\rangle &= \sum_{j_y} |[j_x \otimes (l_y \otimes s_k)_{j_y}]_{JJ_z}\rangle \\ &\times (-1)^{j_x+j_y-J} (-1)^{l_y+s_k+j_x+J} [(2j_y+1)(2\mathcal{S}+1)]^{\frac{1}{2}} \left\{ \begin{matrix} l_y & s_k & j_y \\ j_x & J & \mathcal{S} \end{matrix} \right\}. \end{aligned} \quad (6)$$

Then,

$$R_{1\frac{3}{2},0\frac{1}{2}}^{\frac{1}{2}} = \frac{2\sqrt{2}}{3} \mathcal{R}_{1\frac{1}{2},0\frac{1}{2}}^{\frac{1}{2}} - \frac{1}{3} \mathcal{R}_{1\frac{3}{2},0\frac{1}{2}}^{\frac{1}{2}}, \quad R_{1\frac{1}{2},0\frac{3}{2}}^{\frac{3}{2}} = -\frac{2}{3} \mathcal{R}_{1\frac{1}{2},0\frac{1}{2}}^{\frac{3}{2}} - \frac{\sqrt{5}}{3} \mathcal{R}_{1\frac{3}{2},0\frac{1}{2}}^{\frac{3}{2}} \quad (7)$$

where,  $\mathcal{R}_{l'j',lj}^J$  is a R-matrix in  $jj$ -basis.

### III. TIME REVERSAL VIOLATING POTENTIALS

The most general form of time reversal violating and parity violating part of nucleon-nucleon Hamiltonian in first order of relative nucleon momentum can be written as the sum

of momentum independent and momentum dependent parts,  $H^{T\bar{P}} = H_{stat}^{T\bar{P}} + H_{non-static}^{T\bar{P}}$  [7],

$$H_{stat}^{T\bar{P}} = g_1(r)\boldsymbol{\sigma}_- \cdot \hat{r} + g_2(r)\tau_1 \cdot \tau_2 \boldsymbol{\sigma}_- \cdot \hat{r} + g_3(r)T_{12}^z \boldsymbol{\sigma}_- \cdot \hat{r} \\ + g_4(r)\tau_+ \boldsymbol{\sigma}_- \cdot \hat{r} + g_5(r)\tau_- \boldsymbol{\sigma}_+ \cdot \hat{r} \quad (8)$$

$$H_{non-static}^{T\bar{P}} = (g_6(r) + g_7(r)\tau_1 \cdot \tau_2 + g_8(r)T_{12}^z + g_9(r)\tau_+) \boldsymbol{\sigma}_\times \cdot \frac{\bar{\mathbf{p}}}{m_N} \\ + (g_{10}(r) + g_{11}(r)\tau_1 \cdot \tau_2 + g_{12}(r)T_{12}^z + g_{13}(r)\tau_+) \\ \times \left( \hat{r} \cdot \boldsymbol{\sigma}_\times \hat{r} \cdot \frac{\bar{\mathbf{p}}}{m_N} - \frac{1}{3} \boldsymbol{\sigma}_\times \cdot \frac{\bar{\mathbf{p}}}{m_N} \right) \\ + g_{14}(r)\tau_- \left( \hat{r} \cdot \boldsymbol{\sigma}_1 \hat{r} \cdot (\boldsymbol{\sigma}_2 \times \frac{\bar{\mathbf{p}}}{m_N}) + \hat{r} \cdot \boldsymbol{\sigma}_2 \hat{r} \cdot (\boldsymbol{\sigma}_1 \times \frac{\bar{\mathbf{p}}}{m_N}) \right) \\ + g_{15}(r)(\tau_1 \times \tau_2)^z \boldsymbol{\sigma}_+ \cdot \frac{\bar{\mathbf{p}}}{m_N} \\ + g_{16}(r)(\tau_1 \times \tau_2)^z \left( \hat{r} \cdot \boldsymbol{\sigma}_+ \hat{r} \cdot \frac{\bar{\mathbf{p}}}{m_N} - \frac{1}{3} \boldsymbol{\sigma}_+ \cdot \frac{\bar{\mathbf{p}}}{m_N} \right), \quad (9)$$

where exact form of  $g_i(r)$  depends on the details of particular theory. Here, we consider three different approaches for description of TRIV interactions: meson exchange model, pionless EFT, and pionful EFT.

TRIV meson exchange potential in general involves exchanges of pions ( $J^P = 0^-, m_\pi = 140$  MeV),  $\eta$ -mesons ( $J^P = 0^-, m_\eta = 550$  MeV), and  $\rho$ - and  $\omega$ -mesons ( $J^P = 1^-, m_{\rho,\omega} = 770, 780$  MeV). To derive this potential, we use strong  $\mathcal{L}^{st}$  and TRIV  $\mathcal{L}_{T\bar{P}}$  Lagrangians, which can be written as [8, 9]

$$\mathcal{L}^{st} = g_\pi \bar{N} i \gamma_5 \tau^a \pi^a N + g_\eta \bar{N} i \gamma_5 \eta N \\ - g_\rho \bar{N} \left( \gamma^\mu - i \frac{\chi_V}{2m_N} \sigma^{\mu\nu} q_\nu \right) \tau^a \rho_\mu^a N \\ - g_\omega \bar{N} \left( \gamma^\mu - i \frac{\chi_S}{2m_N} \sigma^{\mu\nu} q_\nu \right) \omega_\mu N, \quad (10)$$

$$\mathcal{L}_{T\bar{P}} = \bar{N} [\bar{g}_\pi^{(0)} \tau^a \pi^a + \bar{g}_\pi^{(1)} \pi^0 + \bar{g}_\pi^{(2)} (3\tau^z \pi^0 - \tau^a \pi^a)] N \\ + \bar{N} [\bar{g}_\eta^{(0)} \eta + \bar{g}_\eta^{(1)} \tau^z \eta] N \\ + \bar{N} \frac{1}{2m_N} [\bar{g}_\rho^{(0)} \tau^a \rho_\mu^a + \bar{g}_\rho^{(1)} \rho_\mu^0 + \bar{g}_\rho^{(2)} (3\tau^z \rho_\mu^0 - \tau^a \rho_\mu^a)] \sigma^{\mu\nu} q_\nu \gamma_5 N \\ + \bar{N} \frac{1}{2m_N} [\bar{g}_\omega^{(0)} \omega_\mu + \bar{g}_\omega^{(1)} \tau^z \omega_\mu] \sigma^{\mu\nu} q_\nu \gamma_5 N, \quad (11)$$

where  $q_\nu = p_\nu - p'_\nu$ ,  $\chi_V$  and  $\chi_S$  are iso-vector and scalar magnetic moments of a nucleon ( $\chi_V = 3.70$  and  $\chi_S = -0.12$ ), and  $\bar{g}_\alpha^{(i)}$  are TRIV meson-nucleon coupling constants. Further,

we use the following values for strong couplings constants:  $g_\pi = 13.07$ ,  $g_\eta = 2.24$ ,  $g_\rho = 2.75$ ,  $g_\omega = 8.25$ .

Meson exchange models from these Lagrangians lead to TRIV potential

$$\begin{aligned}
V_{T\bar{p}} = & \left[ -\frac{\bar{g}_\eta^{(0)} g_\eta m_\eta^2}{2m_N 4\pi} Y_1(x_\eta) + \frac{\bar{g}_\omega^{(0)} g_\omega m_\omega^2}{2m_N 4\pi} Y_1(x_\omega) \right] \boldsymbol{\sigma}_- \cdot \hat{r} \\
& + \left[ -\frac{\bar{g}_\pi^{(0)} g_\pi m_\pi^2}{2m_N 4\pi} Y_1(x_\pi) + \frac{\bar{g}_\rho^{(0)} g_\rho m_\rho^2}{2m_N 4\pi} Y_1(x_\rho) \right] \tau_1 \cdot \tau_2 \boldsymbol{\sigma}_- \cdot \hat{r} \\
& + \left[ -\frac{\bar{g}_\pi^{(2)} g_\pi m_\pi^2}{2m_N 4\pi} Y_1(x_\pi) + \frac{\bar{g}_\rho^{(2)} g_\rho m_\rho^2}{2m_N 4\pi} Y_1(x_\rho) \right] T_{12}^z \boldsymbol{\sigma}_- \cdot \hat{r} \\
& + \left[ -\frac{\bar{g}_\pi^{(1)} g_\pi m_\pi^2}{4m_N 4\pi} Y_1(x_\pi) + \frac{\bar{g}_\eta^{(1)} g_\eta m_\eta^2}{4m_N 4\pi} Y_1(x_\eta) + \frac{\bar{g}_\rho^{(1)} g_\rho m_\rho^2}{4m_N 4\pi} Y_1(x_\rho) + \frac{\bar{g}_\omega^{(1)} g_\omega m_\omega^2}{2m_N 4\pi} Y_1(x_\omega) \right] \tau_+ \boldsymbol{\sigma}_- \cdot \hat{r} \\
& + \left[ -\frac{\bar{g}_\pi^{(1)} g_\pi m_\pi^2}{4m_N 4\pi} Y_1(x_\pi) - \frac{\bar{g}_\eta^{(1)} g_\eta m_\eta^2}{4m_N 4\pi} Y_1(x_\eta) - \frac{\bar{g}_\rho^{(1)} g_\rho m_\rho^2}{4m_N 4\pi} Y_1(x_\rho) + \frac{\bar{g}_\omega^{(1)} g_\omega m_\omega^2}{2m_N 4\pi} Y_1(x_\omega) \right] \tau_- \boldsymbol{\sigma}_+ \cdot \hat{r},
\end{aligned} \tag{12}$$

where  $T_{12}^z = 3\tau_1^z \tau_2^z - \tau_1 \cdot \tau_2$ ,  $Y_1(x) = (1 + \frac{1}{x}) \frac{e^{-x}}{x}$ ,  $x_a = m_a r$ .

Comparing eq. (8) with this potential, one can see that  $g_i(r)$  functions in meson exchange model are defined as

$$\begin{aligned}
g_1^{ME}(r) &= -\frac{\bar{g}_\eta^{(0)} g_\eta m_\eta^2}{2m_N 4\pi} Y_1(x_\eta) + \frac{\bar{g}_\omega^{(0)} g_\omega m_\omega^2}{2m_N 4\pi} Y_1(x_\omega) \\
g_2^{ME}(r) &= -\frac{\bar{g}_\pi^{(0)} g_\pi m_\pi^2}{2m_N 4\pi} Y_1(x_\pi) + \frac{\bar{g}_\rho^{(0)} g_\rho m_\rho^2}{2m_N 4\pi} Y_1(x_\rho) \\
g_3^{ME}(r) &= -\frac{\bar{g}_\pi^{(2)} g_\pi m_\pi^2}{2m_N 4\pi} Y_1(x_\pi) + \frac{\bar{g}_\rho^{(2)} g_\rho m_\rho^2}{2m_N 4\pi} Y_1(x_\rho) \\
g_4^{ME}(r) &= -\frac{\bar{g}_\pi^{(1)} g_\pi m_\pi^2}{4m_N 4\pi} Y_1(x_\pi) + \frac{\bar{g}_\eta^{(1)} g_\eta m_\eta^2}{4m_N 4\pi} Y_1(x_\eta) + \frac{\bar{g}_\rho^{(1)} g_\rho m_\rho^2}{4m_N 4\pi} Y_1(x_\rho) + \frac{\bar{g}_\omega^{(1)} g_\omega m_\omega^2}{2m_N 4\pi} Y_1(x_\omega) \\
g_5^{ME}(r) &= -\frac{\bar{g}_\pi^{(1)} g_\pi m_\pi^2}{4m_N 4\pi} Y_1(x_\pi) - \frac{\bar{g}_\eta^{(1)} g_\eta m_\eta^2}{4m_N 4\pi} Y_1(x_\eta) - \frac{\bar{g}_\rho^{(1)} g_\rho m_\rho^2}{4m_N 4\pi} Y_1(x_\rho) + \frac{\bar{g}_\omega^{(1)} g_\omega m_\omega^2}{2m_N 4\pi} Y_1(x_\omega),
\end{aligned} \tag{13}$$

For TRIV potentials in pionless EFT potential, these functions are

$$\begin{aligned}
g_1^\pi(r) &= \frac{c_1^\pi}{2m_N} \frac{d}{dr} \delta^{(3)}(\mathbf{r}) \rightarrow -\frac{c_1^\pi \mu^2}{2m_N} \frac{\mu^2}{4\pi} Y_1(\mu r) \\
g_2^\pi(r) &= \frac{c_2^\pi}{2m_N} \frac{d}{dr} \delta^{(3)}(\mathbf{r}) \rightarrow -\frac{c_2^\pi \mu^2}{2m_N} \frac{\mu^2}{4\pi} Y_1(\mu r) \\
g_3^\pi(r) &= \frac{c_3^\pi}{2m_N} \frac{d}{dr} \delta^{(3)}(\mathbf{r}) \rightarrow -\frac{c_3^\pi \mu^2}{2m_N} \frac{\mu^2}{4\pi} Y_1(\mu r) \\
g_4^\pi(r) &= \frac{c_4^\pi}{2m_N} \frac{d}{dr} \delta^{(3)}(\mathbf{r}) \rightarrow -\frac{c_4^\pi \mu^2}{2m_N} \frac{\mu^2}{4\pi} Y_1(\mu r) \\
g_5^\pi(r) &= \frac{c_5^\pi}{2m_N} \frac{d}{dr} \delta^{(3)}(\mathbf{r}) \rightarrow -\frac{c_5^\pi \mu^2}{2m_N} \frac{\mu^2}{4\pi} Y_1(\mu r),
\end{aligned} \tag{14}$$

where low energy constants (LECs)  $c_i^\pi$  of pionless EFT have the dimension  $[fm^2]$ . In our calculations with this potential, we use Yukawa function ( $\frac{\mu^3}{4\pi} Y_0(\mu r)$ , where  $Y_0(x) = \frac{e^{-x}}{x}$ ) with regularization scale  $\mu = m_\pi$ , instead of singular  $\delta^{(3)}(r)$  in paper [9].

The pionful EFT acquire long range terms due to the one pion exchange in addition to the short range term expressions equivalent to ones provided by the pionless EFT. Then, ignoring two pion exchange contributions at the middle range and higher order corrections, one can write  $g_i(r)$  functions for the pionful EFT as

$$\begin{aligned}
g_1^\pi(r) &= -\frac{c_1^\pi \mu^2}{2m_N} \frac{\mu^2}{4\pi} Y_1(\mu r) \\
g_2^\pi(r) &= -\frac{c_2^\pi \mu^2}{2m_N} \frac{\mu^2}{4\pi} Y_1(\mu r) - \frac{\bar{g}_\pi^{(0)} g_\pi}{2m_N} \frac{m_\pi^2}{4\pi} Y_1(x_\pi) \\
g_3^\pi(r) &= -\frac{c_3^\pi \mu^2}{2m_N} \frac{\mu^2}{4\pi} Y_1(\mu r) - \frac{\bar{g}_\pi^{(2)} g_\pi}{2m_N} \frac{m_\pi^2}{4\pi} Y_1(x_\pi) \\
g_4^\pi(r) &= -\frac{c_4^\pi \mu^2}{2m_N} \frac{\mu^2}{4\pi} Y_1(\mu r) - \frac{\bar{g}_\pi^{(1)} g_\pi}{4m_N} \frac{m_\pi^2}{4\pi} Y_1(x_\pi) \\
g_5^\pi(r) &= -\frac{c_5^\pi \mu^2}{2m_N} \frac{\mu^2}{4\pi} Y_1(\mu r) - \frac{\bar{g}_\pi^{(1)} g_\pi}{4m_N} \frac{m_\pi^2}{4\pi} Y_1(x_\pi).
\end{aligned} \tag{15}$$

For this potential, the cutoff scale  $\mu$  is larger than pion mass, because pion is a degree of freedom of the theory. Therefore, in general magnitudes of LECs and their scaling behavior, as a function of a cutoff parameter  $c_i^\pi(\mu)$ , are different from  $c_i^\pi(\mu)$  ones.

One can see that all these three potentials which come from different approaches have exactly the same operator structure. The only difference between them is related in different scalar function multiplied by each operator, which, in turn, defer only by different scales of characteristic masses:  $m_\pi$ ,  $m_\eta$ ,  $m_\rho$ , and  $m_\omega$ . Therefore, to unify notations, it is convenient

to define new constants  $C_n^a$  (of dimension of  $[fm]$ ) and scalar function  $f_n^a(r) = \frac{\mu^2}{4\pi} Y_1(\mu r)$  (of dimension of  $[fm^{-2}]$ ) as

$$g_n(r) \equiv \sum_a C_n^a f_n^a(r), \quad (16)$$

where the form of  $C_n^a$  and  $f_n^a(r)$  can be read from eq. (13), (14) and (15).

Since non-static TRIV potentials, with  $g_{n>5}$ , do not appear either in meson exchange model or in the lowest order EFTs, they can be considered as a higher order correction to the lowest order EFT or related to heavy meson contributions in the meson exchange model. Nevertheless, for a completeness of consideration, we estimate the contributions of these operators using  $f_n^a(r)$  functions with proper mass scales.

#### IV. CALCULATION OF TRIV AMPLITUDES

The non-perturbed (parity conserving) 3-body wave functions for neutron-deuteron scattering are obtained by solving Faddeev equations (also often called Kowalski-Noyes equations) in configuration space [10, 11]. The wave function in Faddeev formalism is a sum of three Faddeev components,

$$\Psi(\mathbf{x}, \mathbf{y}) = \psi_1(\mathbf{x}_1, \mathbf{y}_1) + \psi_2(\mathbf{x}_2, \mathbf{y}_2) + \psi_3(\mathbf{x}_3, \mathbf{y}_3). \quad (17)$$

In a particular case of three identical particles (this becomes formally true for three-nucleon system in the isospin formalism), three Faddeev equations (components) become formally identical. By accommodating the three-nucleon force, which under nucleon permutation might be expressed as a symmetric sum of three terms:  $V_{ijk} = V_{ij}^k + V_{jk}^i + V_{ki}^j$ , Faddeev equations read:

$$(E - H_0 - V_{ij}) \psi_k = V_{ij}(\psi_i + \psi_j) + \frac{1}{2}(V_{jk}^i + V_{ki}^j) \psi \quad (18)$$

where  $(ijk)$  are particle indices,  $H_0$  is kinetic energy operator,  $V_{ij}$  is two body force between particles  $i$ , and  $j$ , and  $\psi_k = \psi_{ij,k}$  is Faddeev component.

Using relative Jacobi coordinates  $\mathbf{x}_k = (\mathbf{r}_j - \mathbf{r}_i)$  and  $\mathbf{y}_k = \frac{2}{\sqrt{3}}(\mathbf{r}_k - \frac{\mathbf{r}_i + \mathbf{r}_j}{2})$ , one can expand these Faddeev components in bipolar harmonic basis:

$$\psi_k = \sum_{\alpha} \frac{F_{\alpha}(x_k, y_k)}{x_k y_k} \left| (l_x (s_i s_j)_{s_x})_{j_x} (l_y s_k)_{j_y} \right\rangle_{JM} \otimes \left| (t_i t_j)_{t_x} t_k \right\rangle_{TT_z}, \quad (19)$$

where index  $\alpha$  represents all allowed combinations of the quantum numbers presented in the brackets:  $l_x$  and  $l_y$  are the partial angular momenta associated with respective Jacobi coordinates,  $s_i$  and  $t_i$  are the spins and isospins of the individual particles. Functions  $F_\alpha(x_k, y_k)$  are called partial Faddeev amplitudes. It should be noted that the total angular momentum  $J$  as well as its projection  $M$  are conserved, but the total isospin  $T$  of the system is not conserved due to the presence of charge dependent terms in nuclear interactions.

Boundary conditions for Eq. (18) can be written in the Dirichlet form. Thus, Faddeev amplitudes satisfy the regularity conditions:

$$F_\alpha(0, y_k) = F_\alpha(x_k, 0) = 0. \quad (20)$$

For neutron-deuteron scattering with energies below the break-up threshold, Faddeev components vanish for  $\mathbf{x}_k \rightarrow \infty$ . If  $\mathbf{y}_k \rightarrow \infty$ , then interactions between the particle  $k$  and the cluster  $ij$  are negligible, and Faddeev components  $\psi_i$  and  $\psi_j$  vanish. Then, for the component  $\psi_k$ , which describes the plane wave of the particle  $k$  with respect to the bound particle pair  $ij$ ,

$$\begin{aligned} \lim_{y_k \rightarrow \infty} \psi_k(\mathbf{x}_k, \mathbf{y}_k)_{l_n j_n} &= \frac{1}{\sqrt{3}} \sum_{j'_n l'_n} \left| \{ \phi_d(\mathbf{x}_k) \}_{j_d} \otimes \{ Y_{l'_n}(\hat{\mathbf{y}}_k) \otimes s_k \}_{j'_n} \right\rangle_{JM} \otimes \left| (t_i t_j)_{t_d} t_k \right\rangle_{\frac{1}{2}, -\frac{1}{2}} \\ &\times \frac{i}{2} \left[ \delta_{l'_n j'_n, l_n j_n} h_{l'_n}^-(pr_{nd}) - S_{l'_n j'_n, l_n j_n} h_{l'_n}^+(pr_{nd}) \right], \end{aligned} \quad (21)$$

where deuteron, being formed from nucleons  $i$  and  $j$ , has quantum numbers  $s_d = 1$ ,  $j_d = 1$ , and  $t_d = 0$ , and its wave function  $\phi_d(\mathbf{x}_k)$  is normalized to unity. Here,  $r_{nd} = (\sqrt{3}/2)y_k$  is the relative distance between neutron and deuteron target, and  $h_{l'_n}^\pm$  are the spherical Hankel functions. The expression (21) is normalized to satisfy a condition of unit flux for  $nd$  scattering wave function.

For the cases where Urbana type three-nucleon interaction (TNI) is included, we modify the Faddeev equation (18) into

$$(E - H_0 - V_{ij}) \psi_k = V_{ij}(\psi_i + \psi_j) + \frac{1}{2}(V_{jk}^i + V_{ki}^j) \Psi \quad (22)$$

by noting that the TNI among particles  $ijk$  can be written as the sum of three terms:  $V_{ijk} = V_{ij}^k + V_{jk}^i + V_{ki}^j$ .

Using decomposition of momentum  $\bar{\mathbf{p}}$  which acts only on the nuclear wave function,

$$\bar{\mathbf{p}} = \frac{i \overleftarrow{\nabla}_x - i \overrightarrow{\nabla}_x}{2} = \frac{i \hat{x}}{2} \left( \frac{\overleftarrow{\partial}}{\partial x} - \frac{\overrightarrow{\partial}}{\partial x} \right) + \frac{i}{2} \frac{1}{x} \left( \overleftarrow{\nabla}_\Omega - i \overrightarrow{\nabla}_\Omega \right), \quad (23)$$

we can represent general matrix elements of local two-body parity violating potential operators as

$${}^{(-)}\langle\Psi_f|O|\Psi_i\rangle^{(+)} = \left(\frac{\sqrt{3}}{2}\right)^3 \sum_{\alpha\beta} \left[ \int dx x^2 dy y^2 \left( \frac{\tilde{F}_{f,\alpha}^{(+)}(x,y)}{xy} \right) \hat{X}(x) \left( \frac{\tilde{F}_{i,\beta}^{(+)}(x,y)}{xy} \right) \right] \langle\alpha|\hat{O}(\hat{x})|\beta\rangle, \quad (24)$$

where  $(\pm)$  means outgoing and incoming boundary conditions and  $\hat{X}(x)$  is a scalar function or derivative acting on wave function with respect to  $x$ . (Note that we have used the fact that  $(\tilde{F}^{(-)})^* = \tilde{F}^{(+)}$ .) The partial amplitudes  $\tilde{F}_{i(f),\alpha}(x,y)$  represent the total systems wave function in one selected basis set among three possible angular momentum coupling sequences for three particle angular momenta:

$$\Psi_{i(f)}(x,y) = \sum_{\alpha} \frac{\tilde{F}_{i(f),\alpha}(x,y)}{xy} \left| (l_x (s_i s_j)_{s_x})_{j_x} (l_y s_k)_{j_y} \right\rangle_{JM} \otimes \left| (t_i t_j)_{t_x} t_k \right\rangle_{TT_z}. \quad (25)$$

The “angular” part of the matrix element is

$$\langle\alpha|\hat{O}(\hat{x})|\beta\rangle \equiv \int d\hat{x} \int d\hat{y} \mathcal{Y}_{\alpha}^{\dagger}(\hat{x}, \hat{y}) \hat{O}(\hat{x}) \mathcal{Y}_{\beta}(\hat{x}, \hat{y}), \quad (26)$$

where  $\mathcal{Y}_{\alpha}(\hat{x}, \hat{y})$  is a tensor bipolar spherical harmonic with a quantum number  $\alpha$ . One can see that operators for “angular” matrix elements have the following structure:

$$\hat{O}(\hat{x}) = (\tau_i \odot \tau_j) (\boldsymbol{\sigma}_i \odot \boldsymbol{\sigma}_j) \cdot (\hat{x}, \text{ or } \overleftarrow{\nabla}_{\Omega}, \text{ or } \overrightarrow{\nabla}_{\Omega}), \quad (27)$$

where  $\odot, \otimes = \pm, \times$ . We calculated the “angular” matrix elements by representing all operators as a tensor product of isospin, spin, spatial operators. For details of the calculations of matrix elements, see paper [6]. Similar approaches have been successfully applied for calculations of weak and electromagnetic processes involving three-body and four-body hadronic systems [12–17] and for calculation of parity violating effects in neutron deuteron scattering [6, 18].

## V. RESULTS AND DISCUSSIONS

Typical results for contributions of different operators of a TRIV potential to matrix elements are shown in table I, where a mass scale was chosen to be equal to  $\mu = 138 \text{ MeV}$ . As it was discussed, both pionless and pionfull EFTs in the leading order, as well as the meson exchange model, have only first five operators which have non-zero values. Taking

TABLE I. A typical matrix elements of TRIV potential,  $\text{Re} \frac{\langle (l_y' j_y'), J | V_n^{T\hat{P}} | (l_y j_y), J \rangle}{C_{np}}$ , in jj-coupling scheme with  $AV18 + UIX$  strong potential at zero energy limit. Imaginary part of potential matrix element is zero at zero energy limit. Scalar functions are chosen as  $\frac{m_\pi^2}{4\pi} Y_1(m_\pi r)$  for operators 1 – 5,  $\frac{m_\pi^2}{4\pi} Y_0(m_\pi r)$  for operators 6 – 16.  $O_{3,8,12} = 0$  because of isospin selection rules. All data are in  $fm^2$ .

n	$\langle 1\frac{1}{2}   v^{1/2}   0\frac{1}{2} \rangle / p$	$\langle 1\frac{3}{2}   v^{1/2}   0\frac{1}{2} \rangle / p$	$\langle 1\frac{1}{2}   v^{3/2}   0\frac{1}{2} \rangle / p$	$\langle 1\frac{3}{2}   v^{3/2}   0\frac{1}{2} \rangle / p$
1	$0.590 \times 10^{-01}$	$-0.787 \times 10^{-01}$	$0.151 \times 10^{-01}$	$0.177 \times 10^{-01}$
2	$0.627 \times 10^{+00}$	$-0.863 \times 10^{-01}$	$-0.144 \times 10^{+00}$	$-0.167 \times 10^{+00}$
4	$-0.268 \times 10^{+00}$	$0.107 \times 10^{+00}$	$0.330 \times 10^{-01}$	$0.379 \times 10^{-01}$
5	$0.321 \times 10^{+00}$	$-0.267 \times 10^{+00}$	$-0.199 \times 10^{+00}$	$-0.691 \times 10^{-01}$
6	$0.719 \times 10^{-01}$	$-0.104 \times 10^{-01}$	$-0.115 \times 10^{-01}$	$-0.141 \times 10^{-01}$
7	$-0.206 \times 10^{-01}$	$0.520 \times 10^{-02}$	$0.337 \times 10^{-01}$	$0.384 \times 10^{-01}$
9	$-0.650 \times 10^{-01}$	$0.865 \times 10^{-02}$	$0.238 \times 10^{-03}$	$0.134 \times 10^{-02}$
10	$0.106 \times 10^{-01}$	$-0.932 \times 10^{-03}$	$0.658 \times 10^{-03}$	$0.622 \times 10^{-03}$
11	$0.171 \times 10^{-01}$	$-0.548 \times 10^{-03}$	$-0.237 \times 10^{-02}$	$-0.273 \times 10^{-02}$
13	$-0.163 \times 10^{-01}$	$0.111 \times 10^{-02}$	$0.131 \times 10^{-03}$	$0.288 \times 10^{-03}$
14	$0.649 \times 10^{-02}$	$-0.628 \times 10^{-02}$	$-0.876 \times 10^{-02}$	$-0.250 \times 10^{-03}$
15	$0.338 \times 10^{-01}$	$-0.230 \times 10^{-01}$	$-0.293 \times 10^{-01}$	$-0.198 \times 10^{-02}$
16	$0.128 \times 10^{-01}$	$-0.816 \times 10^{-02}$	$-0.119 \times 10^{-01}$	$-0.335 \times 10^{-03}$

into account that the characteristic mass scale  $\mu$  for operator with  $g_{n \geq 6}$  should be at least larger than two-pion mass (since two pion exchange corresponds to higher order corrections), the actual contributions of these operators are at least one order of magnitude smaller than the value shown in Table I. Thus, one can neglect contributions from the suppressed  $n \geq 6$  operators provided coupling constants satisfy the naturalness assumption.

The possible contributions of different mesons to TRIV amplitude at  $E_{cm} = 100$  keV are summarized in Table II. Using these data, the observable parameters at the neutron energy  $E_{cm} = 100$  keV can be re-written in terms of TRIV meson coupling constants as

$$\begin{aligned} \frac{1}{N} \frac{d\phi^{T\hat{P}}}{dz} = & (-65 \text{ rad} \cdot \text{fm}^2) [\bar{g}_\pi^{(0)} + 0.12\bar{g}_\pi^{(1)} + 0.0072\bar{g}_\eta^{(0)} + 0.0042\bar{g}_\eta^{(1)} \\ & - 0.0084\bar{g}_\rho^{(0)} + 0.0044\bar{g}_\rho^{(1)} - 0.0099\bar{g}_\omega^{(0)} + 0.00064\bar{g}_\omega^{(1)}] \end{aligned} \quad (28)$$

TABLE II. Difference of scattering amplitudes,  $(f_+^{T\mathbb{P}} - f_-^{T\mathbb{P}})/(pC_n)$  for TRIV potential operators  $n = 1, 2, 4$ , and  $5$  for mass scales corresponding to meson masses at  $E_{cm} = 100$  keV. All data are in  $fm$ .

n	$\frac{\Delta f^\pi}{p}$	$\frac{\Delta f^\eta}{p}$	$\frac{\Delta f^\rho}{p}$	$\frac{\Delta f^\omega}{p}$
1	$-0.615 - i0.0567$	$-0.317 - i0.00738$	$-0.125 - i0.00329$	$-0.119 - i0.00317$
2	$-7.58 + i1.07$	$-0.761 + i0.0901$	$-0.302 + i0.0361$	$-0.288 + i0.0345$
4	$3.14 - i0.300$	$0.571 - i0.0227$	$0.225 - i0.00873$	$0.215 - i0.00832$
5	$-4.99 + i0.848$	$-0.262 + i0.0717$	$-0.0934 + i0.0273$	$-0.0888 + i0.0260$

and

$$P^{T\mathbb{P}} = \frac{\Delta\sigma^{T\mathbb{P}}}{2\sigma_{tot}} = \frac{(-0.185 \text{ b})}{2\sigma_{tot}} [\bar{g}_\pi^{(0)} + 0.26\bar{g}_\pi^{(1)} - 0.0012\bar{g}_\eta^{(0)} + 0.0034\bar{g}_\eta^{(1)} - 0.0071\bar{g}_\rho^{(0)} + 0.0035\bar{g}_\rho^{(1)} + 0.0019\bar{g}_\omega^{(0)} - 0.00063\bar{g}_\omega^{(1)}]. \quad (29)$$

For a comparison, DDH model of PV interaction with AV18+UIX strong potential at  $E_{cm} = 100$  keV gives

$$\frac{1}{N} \frac{d\phi^{\mathbb{P}}}{dz} = (55 \text{ rad} \cdot \text{fm}^2) \left[ h_\pi^1 + h_\rho^0(0.11) + h_\rho^1(-0.035) + h_\omega^0(0.14) + h_\omega^1(-0.12) + h_\rho'^1(-0.013) \right] \quad (30)$$

$$P^{\mathbb{P}} = \frac{\Delta\sigma^{\mathbb{P}}}{2\sigma_{tot}} = \frac{(0.395 \text{ b})}{2\sigma_{tot}} \left[ h_\pi^1 + h_\rho^0(0.021) + h_\rho^1(0.0027) + h_\omega^0(0.022) + h_\omega^1(-0.043) + h_\rho'^1(-0.012) \right]. \quad (31)$$

These expressions correspond to

$$\frac{1}{N} \frac{d\phi^{\mathbb{P}}}{dz} = (59 \text{ rad} \cdot \text{fm}^2) \left[ h_\pi^1 + h_\rho^0(0.10) + h_\omega^0(0.14) + h_\rho^1(-0.042) + h_\omega^1(-0.12) + h_\rho'^1(0.014) \right] \quad (32)$$

for at zero energy limit, and to

$$P^{\mathbb{P}} = \frac{\Delta\sigma^{\mathbb{P}}}{2\sigma_{tot}} = \frac{(0.140 \text{ b})}{2\sigma_{tot}} \left[ h_\pi^1 + h_\rho^0(0.021) + h_\omega^0(0.022) + h_\rho^1(0.002) + h_\omega^1(-0.044) + h_\rho'^1(-0.012) \right] \quad (33)$$

at  $E_{cm} = 10$  keV, which were calculated with for DDH-II/AV18+UIX potentials in paper [6]. The equations satisfy the expected dependence of  $\Delta\sigma^{T\mathbb{P}}$  and  $\Delta\sigma^{\mathbb{P}}$  on neutron energy

as  $(E_n)^{1/2}$ . The angle of spin rotation, being proportional to the scattering length, is not sensitive to neutron energy at low energy regime.

The results of Table II also could be considered as an illustration of the cutoff dependence of matrix elements for EFT calculations. However, physical observables do not depend on the cutoff due to the renormalization of  $C_i^{\pi} = -\frac{c_i^{\pi}\mu^2}{2m_N}$ . In pionless EFT with cutoff  $\mu = m_{\pi}$ , observables can be written in terms of dimensional LECs,  $c_i^{\pi}$  (in  $fm^2$ ),

$$\begin{aligned} \frac{1}{N} \frac{d\phi^{T\bar{P}}}{dz} &= (-2.45 \text{ rad})[c_2^{\pi} + c_1^{\pi}(0.081) + c_4^{\pi}(0.41) + c_5^{\pi}(0.66)], \\ P^{T\bar{P}} &= \frac{\Delta\sigma^{T\bar{P}}}{2\sigma_{tot}} = \frac{(-0.35)}{\sigma_{tot}}[c_2^{\pi} + c_1^{\pi}(-0.053) + c_4^{\pi}(-0.28) + c_5^{\pi}(0.79)]. \end{aligned} \quad (34)$$

For the case of pionful EFT, one pion exchange contribution is taken explicitly, and all other cutoffs for contact terms should be larger than pion mass. Therefore, the results in table II for pion,  $\rho$ , and  $\omega$  masses correspond to results for different  $\mu$ 's. For example, choosing cutoff scale  $\mu = m_{\rho}$ , the expressions for TRIV observables are

$$\begin{aligned} \frac{1}{N} \frac{d\phi^{T\bar{P}}}{dz} &= (-65 \text{ rad} \cdot \text{fm}^2)[\bar{g}_{\pi}^{(0)} + 0.12\bar{g}_{\pi}^{(1)}] \\ &\quad + (-3.05 \text{ rad})[c_2^{\pi} + c_1^{\pi}(0.41) + c_4^{\pi}(-0.75) + c_5^{\pi}(0.31)] \end{aligned} \quad (35)$$

and

$$\begin{aligned} P^{T\bar{P}} &= \frac{\Delta\sigma^{T\bar{P}}}{2\sigma_{tot}} = \frac{(-0.185 \text{ b})}{2\sigma_{tot}}[\bar{g}_{\pi}^{(0)} + 0.26\bar{g}_{\pi}^{(1)}] \\ &\quad + \frac{(-0.728)}{2\sigma_{tot}}[c_2^{\pi} + c_1^{\pi}(-0.091) + c_4^{\pi}(-0.24) + c_5^{\pi}(0.76)]. \end{aligned} \quad (36)$$

It should be noted that all existing calculation of TRIV couplings are based on the meson exchange model, since EFT low energy constants for TRIV interactions are unknown. Using meson exchange model, one can predict TRIV effects for different models of CP-violation mechanism, because values of TRIV meson-nucleon coupling constants depend on models of CP-violation.

The results of the calculations show that the dominant contributions to TRIV effects come from the first five operators. Moreover, in meson exchange formalism, pion exchange contribution is dominant, provided that CP-odd coupling constants for all mesons have the same order of magnitude. Thus, comparing Eqs.(28) and (29) with Eqs.(30) and (31), one can see that contributions from  $\rho$  and  $\omega$  mesons to TRIV effects are suppressed by about one order of magnitude in comparison to the contributions of these mesons to PV effects. This

fact is especially interesting because, in the majority of models of CP violation, TRIV pion nucleon coupling constants are much larger than  $\rho$  and  $\omega$  ones (for details see, for example [19–22] and references therein.) Assuming dominant contributions of  $\pi$  -mesons and using the conventional parameter [8, 23]  $\lambda = \bar{g}_\pi/h_\pi^1$ , one can describe the TRIV observable in terms of corresponding PV ones as

$$\begin{aligned}\frac{\phi^{T\cancel{P}}}{\phi^{\cancel{P}}} &\simeq (1.2) \left( \frac{\bar{g}_\pi^{(0)}}{h_\pi^1} + (0.12) \frac{\bar{g}_\pi^{(1)}}{h_\pi^1} \right), \\ \frac{\Delta\sigma^{T\cancel{P}}}{\Delta\sigma^{\cancel{P}}} &\simeq (-0.47) \left( \frac{\bar{g}_\pi^{(0)}}{h_\pi^1} + (0.26) \frac{\bar{g}_\pi^{(1)}}{h_\pi^1} \right).\end{aligned}\tag{37}$$

These ratios of TRIV and PV parameters do not depend on neutron energy.

It is useful to relate these estimates to the existing experimental constrains obtained from electric dipole moment (EDM) measurements, even in the case of model dependent relations. For example, the CP-odd coupling constant  $\bar{g}_\pi^{(0)}$  could be related to the value of neutron electric dipole moment (EDM)  $d_n$  generated via a  $\pi^-$  -loop in the chiral limit [24] as

$$d_n = \frac{e}{4\pi m_N} \bar{g}_\pi^{(0)} g_\pi \ln \frac{\Lambda}{m_\pi},\tag{38}$$

where  $\Lambda \simeq m_\rho$ . Then, using experimental limit [25] on  $d_n$ , one can estimate  $\bar{g}_\pi^{(0)} < 2.5 \cdot 10^{-10}$ . The constant  $\bar{g}_\pi^{(1)}$  can be bounded using constraint [26] on  $^{199}\text{Hg}$  atomic EDM as  $\bar{g}_\pi^{(1)} < 0.5 \cdot 10^{-11}$  [27].

Theoretical predictions for  $\lambda$  can vary from  $10^{-2}$  to  $10^{-10}$  for different models of CP violations (see, for example, [8, 19–21, 23] and references therein). Therefore, one can estimate a range of possible values of TRIV observable and relate a particular mechanism of CP-violation to their values. It should be noted that the above parametrization assumes that pion meson exchange contribution is dominant for PV effects. Should the  $\vec{n} + p \rightarrow d + \gamma$  experiment confirm the “best value” of the DDH pion-nucleon coupling constant  $h_\pi^1$ , Eqs.(37) can be considered as an estimate for the value of TRIV effects in neutron-deuteron scattering. Otherwise, if  $h_\pi^1$  is small, one needs to use  $h_\rho$  or  $h_\omega$  with corresponding weights, which will increase relative values of TRIV effects.

## ACKNOWLEDGMENTS

This work was supported by the DOE grants no. DE-FG02-09ER41621. This work was granted access to the HPC resources of IDRIS under the allocation 2009-i2009056006 made

by GENCI (Grand Equipement National de Calcul Intensif). We thank the staff members of the IDRIS for their constant help.

- 
- [1] V. P. Gudkov, Phys. Rept., **212**, 77 (1992).
  - [2] V. E. Bunakov and V. P. Gudkov, Nucl. Phys., **A401**, 93 (1983).
  - [3] L. Stodolsky, Nucl. Phys., **B197**, 213 (1982).
  - [4] P. K. Kabir, Phys. Rev., **D25**, 2013 (1982).
  - [5] D. A. Varshalovich, A. N. Moskalev, and V. K. Khersonskii, *Quantum Theory of Angular Momentum* (World Scientific, 1988).
  - [6] Y.-H. Song, R. Lazauskas, and V. Gudkov, Phys. Rev., **C83**, 015501 (2011).
  - [7] P. Herczeg, Nucl. Phys., **75**, 655 (1966).
  - [8] P. Herczeg, In Tests of Time Reversal Invariance in Neutron Physics, edited by N. R. Roberson, C. R. Gould and J. D. Bowman (World Scientific, Singapore, 1987), p.24.
  - [9] C. P. Liu and R. G. E. Timmermans, Phys. Rev., **C70**, 055501 (2004).
  - [10] L. D. Faddeev, Sov. Phys. JETP, **12**, 1014 (1961).
  - [11] R. Lazauskas and J. Carbonell, Phys. Rev., **C70**, 044002 (2004).
  - [12] Y.-H. Song, R. Lazauskas, T.-S. Park, and D.-P. Min, Phys. Lett., **B656**, 174 (2007).
  - [13] Y.-H. Song, R. Lazauskas, and T.-S. Park, Phys. Rev., **C79**, 064002 (2009).
  - [14] R. Lazauskas, Y.-H. Song, and T.-S. Park, (2009), arXiv:0905.3119 [nucl-th].
  - [15] T. S. Park *et al.*, Phys. Rev., **C67**, 055206 (2003).
  - [16] S. Pastore, L. Girlanda, R. Schiavilla, M. Viviani, and R. B. Wiringa, Phys. Rev., **C80**, 034004 (2009).
  - [17] L. Girlanda *et al.*, Phys. Rev. Lett., **105**, 232502 (2010).
  - [18] R. Schiavilla, M. Viviani, L. Girlanda, A. Kievsky, and L. E. Marcucci, Phys. Rev., **C78**, 014002 (2008).
  - [19] V. P. Gudkov, In Parity and time reversal violation in compound nuclear states and related topics, edited by N. Auerbach and J. D. Bowman (World Scientific, Singapore, 1995), p.231.
  - [20] V. P. Gudkov, Z.Phys., **A343**, 437 (1992).
  - [21] V. P. Gudkov, X.-G. He, and B. H. McKellar, Phys.Rev., **C47**, 2365 (1993).
  - [22] M. Pospelov, Phys. Lett., **B530**, 123 (2002).

- [23] V. P. Gudkov, Phys.Lett., **B243**, 319 (1990).
- [24] M. Pospelov and A. Ritz, Annals Phys., **318**, 119 (2005).
- [25] C. A. Baker *et al.*, Phys. Rev. Lett., **97**, 131801 (2006).
- [26] M. V. Romalis, W. C. Griffith, and E. N. Fortson, Phys. Rev. Lett., **86**, 2505 (2001).
- [27] V. Dmitriev and I. Khriplovich, Phys.Rept., **391**, 243 (2004).